

Spin polarised scanning tunneling probe for helical Luttinger liquids

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We propose a three terminal spin polarized STM setup for probing the helical nature of the Luttinger liquid edge state that appears in the quantum spin Hall system. We show that the three-terminal tunneling conductance strongly depends on the angle (θ) between the magnetization direction of the tip and the local orientation of the electron spin on the edge while the two terminal conductance is independent of this angle. We demonstrate that chiral injection of an electron into the helical Luttinger liquid (which occurs when θ is zero or π) is associated with fractionalization of the spin of the injected electron in addition to the fractionalization of its charge. We also point out a spin current amplification effect induced by the spin fractionalization.

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Introduction :- A new class of insulators have recently emerged called quantum spin Hall insulators which have gapless edge states due to the topological properties of the band structure [1]. For a two-dimensional insulator, a pair of one-dimensional counter propagating modes appear on the edges [1, 2] which are transformed into helical Luttinger liquids (HLL) due to inter-mode Coulomb interactions [3]. Various aspects of this state [4–9] have been studied. The central point about the HLL is the fact that the spin orientation of the edge electrons, which is dictated by the bulk physics, is correlated with the direction of motion of the electron - *i.e.*, opposite spin modes counter propagate. The existence of such edge channels have already been detected experimentally in a multi-terminal Hall bar setup [10]. But although this experiment does confirm the existence of counter propagating one-dimensional (1-D) modes at the edge, it is not a direct observation of the spin degree of freedom. A central motivation of this letter is to suggest a setup wherein the structure of the spin degree of freedom on the edge can be directly probed.

Motivated by the spin valve (SV) effect, the first idea to probe the spin degree of freedom, would be to replace one of the ferromagnetic leads in a magnetic tunnel junction by the HLL and measure the magneto-resistance, as a function of the relative spin orientation of the HLL and the magnetization direction of the ferromagnetic lead. However, the angle dependent tunnel resistance for the SV depends directly on the degree of polarization of the two leads. For HLL, although the edge modes have a specific spin orientation locally, they have no net polarization, and hence the tunnel resistance would be independent of the spin polarization of the ferromagnetic lead.

In this letter, we show that switching to a three terminal geometry involving a magnetized scanning tunneling microscope (STM) tip facilitates the detection of the spin

orientation of the edge electron by inducing a finite three terminal magneto-resistance. For a normal LL, it is not possible to inject an electron with a well-defined momentum (left or right movers) at a localized point in the wire, and hence extended wires were used as injectors in Ref. 11 to achieve chiral injection. But for HLL, since the direction of motion is correlated with the spin projection, chiral injection (*i.e.*, injecting only left movers or right movers) is possible even at a localized point in the wire. One just needs to tune the direction of polarization of the STM parallel (anti-parallel) to the polarization direction of the edge. Once this is achieved, injection of an upspin (downspin) electron is equivalent to injecting right (left) movers. Hence the HLL has a natural advantage over a normal LL for chiral injection. As was experimentally demonstrated in Ref. 11, chiral injection of electrons can lead to an asymmetry in the currents measured on both sides of the injection region, which is further modified by the LL interaction. In this letter, we show that in a similar setup, the left-right current asymmetry in the wire when voltage biased with respect to the STM has also a strong θ dependence due to the interaction induced scattering of electrons between the right (spin up) and left (spin down) moving edges. For purely chiral injection ($\theta = 0, \pi$), we find that the fraction of the total tunneling current measured at the left and right of the injection region is asymmetric and is given by the splitting factor (left) $A_{c1} = (1 \mp K)/2$ and (right) $A_{c2} = (1 \pm K)/2$ (where K is the LL parameter and the top and bottom signs are for $\theta = 0$ and π respectively) in agreement with the results in Ref. 12, and is a manifestation of charge fractionalization of the injected electron. Observing the asymmetry with a spin polarized STM as a local injector would be an indisputable sign of the helical nature of the edge states, since for the usual LL, no such current asymmetry would be expected for local injection.

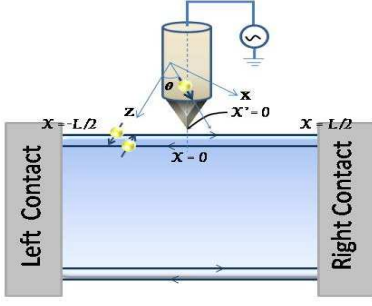


FIG. 1: A schematic of the geometry of the proposed setup. The direction of orientation of the electron spin in the HLL is along the \hat{Z} axis. The angle between direction of orientation of the spin of electrons in the edge and the majority spin in the STM tip is θ and they are assumed to lie in the \hat{X} - \hat{Z} plane. The \hat{Y} -axis points out of the plane of the paper. Here x and x' represent the intrinsic one dimensional coordinates of the STM tip and the wire.

The most subtle outcome of our analysis is the fractionalization of the spin of the injected electron. In contrast to charge fractionalization, the spin gets fractionalized such that one of the fractionalized components turns out to be larger than the injected spin. The asymmetric fractions of the total injected electron spin current are given by $A_{s1} = (1 \mp K)/4K$ and $A_{s2} = (1 \pm K)/4K$ (upper and lower signs for $\theta = 0$ and $\theta = \pi$ respectively) and are a manifestation of the fractionalization of the injected electron spin in the HLL. Note that for $K < 1$ (repulsive electrons) $A_{s2} > 1/2$, thus resulting in an effective magnification of the injected spin current at the right lead.

Geometry :- We propose a three terminal junction as shown in Fig. 1. Three terminal setups have also been used to study tunneling into a quantum wire in the Fabry-Perot regime[13]. The spin of the electrons in the edge states are polarized in some direction depending on details of the spin-orbit interaction in the bulk. We use a coordinate system which has its \hat{Z} -axis along the direction of orientation of the spin of the edge electrons and the plane containing the polarization direction of the edge electron and the tip electron is assumed to be the \hat{X} - \hat{Z} plane (see Fig. 1). Note that here we have assumed that the edge is smooth and is along a straight line, so that there is a well defined quantization direction for the electron spin living on the edge.

The Hamiltonian for the HLL is given by [5]

$$H_0 = v \int_{-L/2}^{L/2} dx \left[K(\partial_x \Phi)^2 + K^{-1}(\partial_x \Theta)^2 \right], \quad (1)$$

where $\Phi = (\phi_{R\uparrow} + \phi_{L\downarrow})/2$, $\Theta = (\phi_{R\uparrow} - \phi_{L\downarrow})/2$ and the $\phi_{R\uparrow/L\downarrow}$ are related to the up and down electron operators in the edge by the standard bosonization identity $\psi_{R\uparrow}(x) \sim \frac{1}{\sqrt{2\pi\zeta}} e^{ik_F x} e^{i\phi_{R\uparrow}(x)}$, $\psi_{L\downarrow}(x) \sim \frac{1}{\sqrt{2\pi\zeta}} e^{-ik_F x} e^{i\phi_{L\downarrow}(x)}$. ζ and K are the short distance cut-

off and the Luttinger parameter respectively. Unlike the standard LL, here the spin orientation is correlated with the direction of motion. We drop Klein factors as they are irrelevant for our computations.

The Hamiltonian for the STM is assumed to be that of a free electron in 1-D. The tunneling Hamiltonian between the tip and the helical edge at a position $x = 0$, $x' = 0$ is given by

$$H_t = t \left[\psi_{i\alpha}^\dagger(x=0) \chi_\alpha(x'=0) + \text{h.c.} \right], \quad (2)$$

where $i = R, L$ denotes right and left movers and α denotes the spin index, $\psi_{i\alpha}$ and χ_α denote the electron destruction operator in the HLL and the STM respectively. Voltage bias in the tunneling operator can be introduced simply by replacing $\chi_\alpha(x) \rightarrow \chi_\alpha(x) e^{-iVt/\hbar}$. We will, henceforth, drop the index i, j denoting the direction of motion.

Since the tunneling conserves spin, using a fully polarized STM with polarization direction tuned along the positive or negative direction of \hat{Z} -axis will naturally allow for chiral injection *i.e.*, injecting only right (\uparrow) or left (\downarrow) movers. In the absence of interactions in the HLL, the chirally injected electron will cause both charge current and spin current to flow only to the right or to the left lead, hence leading to a left-right asymmetry. In the presence of interactions in the HLL, due to Coulomb scattering between the right and left movers, the chirally injected charge and spin degrees of freedom of the electron get fractionalized and move in both directions; however, in general, the left-right asymmetry still survives.

Now, let us consider the fully polarized STM tip with the polarization direction making an arbitrary angle θ with respect to the spin of the HLL electron. In the quantization basis of the HLL spins, the tip spinor can be written as $\chi_{\text{rot}} = e^{-i\theta\sigma \cdot \hat{Y}/2} \chi_T$, where χ_T is the tip spinor in a basis where the spin quantization axis is along the STM polarization direction *i.e.*, $\chi_T = (\chi_\uparrow, 0)$. So $\chi_{\text{rot}\uparrow} = \cos(\theta/2) \chi_\uparrow$ and $\chi_{\text{rot}\downarrow} = \sin(\theta/2) \chi_\uparrow$. In other words, the electron in the tip has both \uparrow and \downarrow spins, but the effective tunnel amplitudes are asymmetric (except when $\theta = \pi/2$) and hence, the current asymmetry survives. As a function of the rotation angle θ , the chiral injection goes from being a pure right-mover at $\theta = 0$ to a pure left mover at $\theta = \pi$.

Charge current :- The tunneling Hamiltonian can now be rewritten in terms of χ_\uparrow as

$$H_t = \left[t_\uparrow \psi_\uparrow^\dagger \chi_\uparrow + t_\downarrow \psi_\downarrow^\dagger \chi_\uparrow + \text{h.c.} \right], \quad (3)$$

where $t_\uparrow = t \cos(\theta/2)$ and $t_\downarrow = t \sin(\theta/2)$ can be tuned by tuning θ . The Bogoliubov fields $\tilde{\phi}_{L/R}$ which move unhindered to right and left direction (henceforth we call them the right chiral and left chiral fields) are given by

$$\phi_{\uparrow/\downarrow} = \frac{1}{2\sqrt{K}} \left[(1 \pm K) \tilde{\phi}_R + (1 \mp K) \tilde{\phi}_L \right]. \quad (4)$$

Note that the total electron density on the HLL wire can be expressed in terms of the chiral fields as $\rho(x) = (\sqrt{K}/2\pi)\partial_x\tilde{\phi}_R - (\sqrt{K}/2\pi)\partial_x\tilde{\phi}_L$ thus defining the chiral right (left) densities and the corresponding number operators as

$$\tilde{N}_{R/L} = \int_{-L/2}^{L/2} dx \tilde{\rho}(x)_{R/L} = \pm \frac{\sqrt{K}}{2\pi} \int_{-L/2}^{L/2} dx \partial_x (\tilde{\phi}_{R/L}) . \quad (5)$$

Next we define the operator corresponding to the chiral decomposition of the total charge current as $I_{t\alpha} = d\tilde{N}_\alpha/dt = -i[\tilde{N}_\alpha, H_t]$, where we have set $\hbar = 1$ and electron charge $e = 1$ and $\alpha = R/L$. Using the standard commutation relations of chiral fields, $[\tilde{\phi}_{\uparrow/\downarrow}(x), \tilde{\phi}_{\uparrow/\downarrow}(x')] = \pm i\pi \text{sgn}(x - x')$ the chiral currents can be found to be

$$I_{tR/L}(\theta) = \frac{1}{2} [(1 \pm K) \cos(\theta/2) I_t(\theta = 0) + (1 \mp K) \sin(\theta/2) I_t(\theta = \pi)] . \quad (6)$$

$I_t(\theta) = I_{tL}(\theta) + I_{tR}(\theta)$ is the total tunneling charge current operator for an arbitrary value of θ and $I_t(\theta = 0/\pi) = it(\chi_{\uparrow}^\dagger\psi_{\uparrow/\downarrow} - \psi_{\uparrow/\downarrow}^\dagger\chi_{\uparrow})$. The expectation values of the currents operator in linear response is given by

$$\langle I_t(\theta) \rangle = -\frac{i}{\hbar} \int_{-\infty}^0 d\tau \langle [I_t(\theta, \tau = 0), H_t(\tau)] \rangle . \quad (7)$$

Since the HLL Hamiltonian is left-right symmetric in the absence of the tip and the tip is fully polarised, the value is equal for $\theta = 0$ and $\theta = \pi$ and given by $\langle I_t(\theta = 0) \rangle = \langle I_t(\theta = \pi) \rangle = I_0$. Using the well-known correlation function of LL liquid at finite temperature T , we find

$$I_0 = \frac{e^2}{h} |t|^2 \frac{(T/\Lambda)^\nu}{(\hbar v_F)^2 \Gamma(\nu + 1)} \times V , \quad (8)$$

where Λ is an ultra-violet cutoff and ν is the Luttinger tunneling exponent given by $\nu = -1 + (K + K^{-1})/2$. Here we have assumed that $T \gg T_L, T_V$, where T_L is the temperature equivalent of the length of the wire defined by $v/L = k_B T$ and $T_V = eV/k_B$, is the temperature equivalent of bias voltage.

Using these values, we now obtain the current heading to the right and left ends of the wire as

$$\langle I_{tR,L}(\theta) \rangle = \frac{(1 \pm K \cos \theta)}{2} I_0 . \quad (9)$$

Note that even though the left and right chiral currents which will be measured at the right and left contact depend on θ , the total tunneling current $I_t(\theta) = I_{tL}(\theta) + I_{tR}(\theta)$ is independent of θ . Thus we show that unlike the two terminal tunnel current, the three terminal current is clearly not independent of θ . This is one of the key results of this letter.

Spin currents :- The isolated HLL, even in equilibrium, has a persistent spin current because of the correlation

of the direction of spin with the direction of motion but no charge current. However, here we would like compute the excess spin current that is caused by the inflow of electrons from the STM tip into the edge mode. Now the tunneling induced magnetization of the edge state can be defined as $\mathbf{S} = \int_{-L/2}^{L/2} dx \mathbf{s}(x) = \int_{-L/2}^{L/2} dx (\psi_\alpha^\dagger \vec{\sigma}_{\alpha\beta} \psi_\beta/2)$ where $\mathbf{s}(x)$ is the local spin density. Hence the spin current can be defined as $d\mathbf{S}/dt = -i[\mathbf{S}, H_t]$. Now using bosonization, it is straight-forward to evaluate the \hat{X} , \hat{Y} and \hat{Z} components of the spin current operator as given below

$$\begin{aligned} \dot{S}_X(\theta) &= \frac{1}{2} [\cos(\theta/2) I_t(\theta = \pi) + \sin(\theta/2) I_t(\theta = 0)] , \\ \dot{S}_Y(\theta) &= \frac{1}{2} [\cos(\theta/2) H_t(\theta = \pi) - \sin(\theta/2) H_t(\theta = 0)] , \\ \dot{S}_Z(\theta) &= \frac{1}{2K} [I_{tR}(\theta) - I_{tL}(\theta)] . \end{aligned} \quad (10)$$

Note that \dot{S}_X and \dot{S}_Z are expressible in terms of the current operator while the \dot{S}_Y is expressible only in terms of the tunnel Hamiltonian given in Eq. 3. The difference is related to the fact that only the \hat{X} and \hat{Z} components of the spin are relevant as the injected electron spin from the STM has no component along the \hat{Y} direction. Hence \dot{S}_Y is expected to be zero and indeed the expectation value of \dot{S}_Y is easily seen to be zero, since H_t is left-right symmetric. Now using Eqs. 6, 9 and 10, we get the following expressions (within linear response) for the spin currents towards the left and right contacts -

$$\dot{\mathbf{S}}_{R/L}(\theta) = \left[\frac{K \mp \cos \theta}{2K \sin \theta} \hat{Z} \pm \frac{1}{2K} \hat{X} \right] I_{tR/L}(\theta) . \quad (11)$$

Hence, for arbitrary values of θ , the spin current collected at the right and the left contacts are asymmetric. Now using Eqs. 9 and 11, it is easy to check that total injected spin current

$$\langle \frac{d\mathbf{S}}{dt} \rangle = (\hat{Z} \cos \theta + \hat{X} \sin \theta) \frac{I_0}{2} , \quad (12)$$

is pointing exactly along the magnetization direction of STM as expected.

Charge and spin fractionalization :- Recently, the issue of charge fractionalization has been addressed both theoretically and experimentally in Refs. 11, 12 and 13. The fractionalization of a chirally injected electron charge into the HLL at a point ($x = 0$) can be understood by considering the following commutator

$$[\tilde{\rho}_{R/L}(x), \psi_R^\dagger(0)] = \frac{1 \pm K}{2} \delta(x) \psi_R^\dagger(0) . \quad (13)$$

This implies that the creation of a single right moving electron at $x = 0$ creates simultaneously an excitation of charge $(1 \pm K)/2$ in the right and left going chiral densities, thus leading to fractionalization of electron. (A similar equation (with an overall sign change) works for

the left-movers). Note that the splitting of the total tunneling current into its chiral components (see Eq. 6) is exactly consistent with the splitting of the electron charge. Hence measuring the chiral currents can provide information about the charge fractionalization, as demonstrated in Ref. 11.

Similarly, to study spin fractionalization, we bosonize the \hat{Z} -component of spin density given by $s_Z(x) = (1/2)(\psi_\uparrow^\dagger(x)\psi_\uparrow(x) - \psi_\downarrow^\dagger(x)\psi_\downarrow(x))$ to obtain $s_Z(x) = (1/2K)(\tilde{\rho}_R(x) - \tilde{\rho}_L(x))$. This defines $s_{Z,R/L} = \pm(1/2K)\tilde{\rho}_{R/L}(x)$. Now let us consider the following commutator

$$\left[s_{Z,R/L}(x), \psi_R^\dagger(0) \right] = \frac{1}{2} \left(\frac{1 \pm K}{2K} \right) \delta(x) \psi_R^\dagger(0). \quad (14)$$

This implies that the creation of a single right moving electron at $x = 0$ creates simultaneously spin excitations of spin $(1 \pm K)/2K$ (in units of electronic spin quanta) in the right and left going chiral spin densities, thus leading to K dependent fractionalization of the spin of the injected electron. Now let us consider the \hat{Z} -component of the spin current operator given in Eq. 10. This operator can be chirally decomposed as follows -

$$\langle \dot{S}_Z(\theta) \rangle_{R/L} = \pm \frac{\langle I_{R/L}(\theta) \rangle}{2K} = \pm \left(\frac{1 \pm K \cos \theta}{2K} \right) \frac{I_0}{2}. \quad (15)$$

For chiral injection (*i.e.*, $\theta = 0, \pi$) we note the splitting of the total tunneling spin current ($I_0/2$) into its chiral components is given precisely by $(1 \pm K)/2K$, which is exactly consistent with the splitting of the electron spin evaluated from the commutator. Intriguingly, one of the splitting fractions, $(1 + K)/2K$ is larger than unity for $K < 1$ (*i.e.*, for repulsive electrons). Hence in the three terminal geometry one obtains an interaction ($K \neq 1$) induced amplification of the injected spin current.

Discussion :- Regarding the application of our work to realistic systems, we first point out that our work is directly applicable to edge states in graphene with a small spin-orbit coupling[14] and to other genuine quantum spin Hall insulators like the model considered in Ref. 15 and Bi [16]. However, for $HgTe/CdTe$ quantum wells, where the spin projections actually refer to pseudospin related to the two block diagonal parts of the effective Hamiltonian written in the $|E_1, m_J = +1/2\rangle$, $|H_1, m_J = +3/2\rangle$, $|E_1, m_J = -1/2\rangle$ and $|H_1, m_J = 3/2\rangle$ basis[17], we need to modify our computations. Fig. 1 is still applicable with the \hat{Z} -axis now referring to the crystal growth axis of the quantum well. But the $\psi_{\uparrow/\downarrow}$ states that we have considered in Eq.3 are no longer right or left movers even before interactions have been introduced. We need to introduce the right and left moving fields as $\eta_{\uparrow/\downarrow}$ given by $\psi_\uparrow = a\eta_\uparrow + b\eta_\downarrow$ and $\psi_\downarrow = a'\eta_\uparrow + b'\eta_\downarrow$ where a, b, a', b' are material dependent parameters which denote how the pseudospin states are related to the real spin of the electron. Hence, the tunneling Hamiltonian in Eq.3 can be

rewritten as

$$H_t = \left[\{ (t_\uparrow a + t_\downarrow a') \eta_\uparrow^\dagger + (t_\uparrow b + t_\downarrow b') \eta_\downarrow^\dagger \} \chi_\uparrow + \text{h.c.} \right]. \quad (16)$$

We get pure right-moving or left-moving currents at $\tan \theta/2 = -a'/a$ or at $\tan \theta/2 = -b'/b$. Note that the angle at which the left-moving current disappears is not exactly opposite to the angle at which the right-moving current vanishes, since the real spin of the left-movers and right-movers need not be equal and opposite. With interactions, it is the $\eta_{\uparrow/\downarrow}$ fields which are bosonised and the rest of the formulation goes through provided that the non-interacting reference angles (*i.e.*, the coefficients a, b, a', b') are known. But determining both a, b, a', b' and K is a non-trivial problem. However, if the experiment could be carried out at different temperatures at fixed θ , then since the current I_0 (defined in Eq.8) depends only on K and not on θ , it may be possible in principle (albeit difficult in practise) to extract the value of K from the power law dependence of current. Moreover, the edge states could be known (from other experiments) to be in the weakly interacting regime ($K = 1$). In these cases, this setup can be used to extract the values of the coefficients a, b, a', b' .

Conclusion :- To summarise, in this letter, we have proposed a three-terminal polarised STM set-up as a probe for HLL. We suggest that the spin polarized tip can facilitate local chiral injection. This leads to current asymmetries, with specific θ dependence, whose measurement can lead to undisputed confirmation of the helical nature of the edge state. Chiral injection of the electron into the HLL is also shown to be directly related to the physics of fractionalization of the injected electron spin in addition to the fractionalization of its charge. We also point out that spin fractionalization leads to a spin current amplification effect in the three terminal geometry.

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